Q1. The programming problem
Max $Z = 2x + 3y$ subject to the conditions
$0 \leq x \leq 3, 0 \leq y \leq 4$ is:

(a) not an LPP
(b) an LPP, WI un unbounded feasible region and no solution
(c) an LPP, and Max Z= 18, $x = 3, y = 4$
(d) an LPP, and Max Z = 12, at $x = 0, y = 4$

Q2. The region represented by the system of inequalities $x, y \geq 0;\ -2x + y \leq 4; x + y \geq 3$ and $x - 2y \leq 2$ is

(a) unbounded in first quadrant
(b) unbounded in first and second quadrant
(c) bounded in first quadrant
(d) not feasible

Q3. A linear programming problem is as follows:
Maximize/minimize objective function $z = 2x - y + 5$ subject to constraints.
$3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0.$
If the corner points of feasible region are A(0, 10) B(12, 6) C(20, 0), 0(0, 0), then which of following is true.

(a) Maximum value of z is 40
(b) Minimum value of z is -5
(c) Difference of maximum and minimum values of z is 35
(d) At two corner points value of z are equal.

Q4. The value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$

(d) $\frac{\pi}{3}$

Q5. Match the following column

Column I	Column II
A. If A be any given square matrix of order n, then	I. $A(adj\ A) = (adj\ A)A = \lvert A\rvert I,$
B. A square matrix A is said to be singular	II. $\lvert A\rvert = 0$
C. A square matrix A is said to be non-singular	III. A is non - singular matrix

D. A square matrix A is invertible if and only if A	IV. $\lvert A\rvert \neq 0$

(a) A→I, B→III, C→IV, D→II

(b) A→IV, B→III, C→I, D→II

(c) A→IV, B→II, C→I, D→III

(d) A→I, B→II, C→IV, D→III

Q6. If two vectors are $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$, then the value of $\vec{a} \times \vec{b}$ is

(a) $5\hat{\imath} + \hat{\jmath} - 4\hat{k}$

(b) $3\hat{\imath} + \hat{\jmath} - 4\hat{k}$

(c) $5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

(d) $5\hat{\imath} + \hat{\jmath} + 4\hat{k}$

Q7. The value of integral $\int \frac{dx}{\sqrt{16-9x^2}}$ is

(a) $\sin^{-1}\frac{3x}{4} + C$

(b) $\frac{1}{3}\sin^{-1}\frac{3x}{4} + C$

(c) $\frac{1}{3}\sin^{-1}\frac{x}{4} + C$

(d) $\frac{1}{2}\sin^{-1}\frac{3x}{2} + C$

Q8. Let A be a square matrix of order 4 with $|A| = 8$. If $|\text{adj (adj } (3A))| = 2^m \cdot 3^n$. Then, find the value of $m + n$

(a) 30

(b) 63

(c) 95

(d) 150

Q9. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, what is the angle between $\vec{a}$ and $\vec{b}$?

(a) 0

(b) $\frac{\pi}{2}$

(c) π

(d) $\frac{\pi}{3}$

Q10. Consider the differential equation:

$$(x^2+1)\frac{dy}{dx}+2xy=4x^2$$

Which of the following statements are true regarding the given differential equation?

(A) The differential equation is linear.

(B) The integrating factor of the differential equation is (x^2+1).

(C) The order of the differential equation is 2.

(D) The general solution can be expressed as $y=\frac{4}{3}x+\frac{C}{x^2+1}$.

Options:
(a) A and B only
(b) A, B, and D only
(c) A and D only
(d) B, C, and D only

Solutions:

S1. Ans. (c)
Sol. Given L.P.P.
$z=2x+3y$
$0\le x\le 3$
$0\le y\le 4$
Given constraints form a rectangle.
Corner points are $(0,0)$, $(3,0)$, $(0,4)$ & $(3,4)$.
Maximum value of $z=2(3)+3(4)=6+12=18$
Maximum value of z $=18$ at $(3,4)$.

S2. Ans. (a)
Sol. Given inequalities are
$x,y\ge 0;\ -2x+y\le 4; x+y\ge 3$ and $x-2y\le 2$
We have
$-2x+y=4$ ………… (i)
$x+y=3$ ………….. (ii)
$x-2y=2$ ………….. (iii)
From eq. (i) we get
$(-2,0)$ & $(0,4)$
From eq. (ii), we get
$(3,0)$ & $(0,3)$
From eq. (iii), we get
$(2,0)$ & $(0,-1)$
The common region is unbounded in first quadrant.

S3. Ans. (b)
Sol. Given
$Z=2x-y+5$
Corner points of feasible region are (0, 10), (12, 6), (20, 0) & (0, 0).
At (0, 10) we have $z=2(0)-10+5=-5$ (Minimum)
At (12, 6) we have $z=2(12)-6+5=24-1=23$
At (20, 0) we have $z=2(20)-0+5=45$ (Maximum)
At (0, 0) we have $z=2(0)-0+5=5$

S4. Ans. (d)

Sol. We have

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$= \tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\sin\frac{\pi}{3}\right]$$

$$= \tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$$

$$= \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

S5. Ans. (d)

Sol. If A be any given square matrix of order n, then $A(adj\ A) = (adj\ A)A = |A|I$

A square matrix A is said to be singular $|A| = 0$

A square matrix A is said to be non-singular

$|A| \neq 0$

A square matrix A is invertible if and only if A is non- singular matrix.

S6. Ans. (a)

Sol. We have

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$$

S7. Ans. (b)

Sol. We have

$$I = \int \frac{dx}{16 - 9x^2} = \frac{1}{3}\int \frac{dx}{\sqrt{\frac{16}{9} - x^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{4} + C$$

S8. Ans. (b)

Sol. Since,

$$|\text{adj.(adj. A}| = |\text{A}|^{(n-1)^2}$$

$$|adj(adj\ 3A)| = |3A|^{3^2} = (3^4|\text{ A}|)^9$$

$$= 3^{36}8^9 = 3^{36}(2^3)^9$$

$$= 3^{36}2^{27}$$

On comparing $2^m 3^n = 2^{27}3^{36}$

$m = 27, n = 36$

Now,

$m + n = 27 + 36 = 63$

S9. Ans. (b)

Sol. We have
$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Squaring both sides:

$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$

$$\begin{aligned} |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} &= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} \\ 4\vec{a}\cdot\vec{b} &= 0 \\ \vec{a}\cdot\vec{b} &= 0 \end{aligned}$$

Since $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\,\theta = 0$, and neither vector is null, we get

$\cos\,\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

S10. Ans. (a)
Sol. (A) The given equation can be rewritten in the standard linear form:

$$\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{4x^2}{x^2+1}$$

So, the equation is first-order linear.

(B) The integrating factor (IF) is:

$$e^{\int \frac{2x}{x^2+1}dx} = e^{\ln\,(x^2+1)} = x^2 + 1$$

Hence, IF = $(x^2 + 1)$.

(C) The highest derivative present is dy/dx, so the order is 1, not 2.

(D) Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{1+x^2}\mathrm{y} = \frac{4x^2}{1+x^2}$$

IF = $(x^2 + 1)$

Solution is

$$y(x^2+1) = \int \left\{ \frac{4x^2}{1+x^2} \times (x^2+1) \right\} dx$$

$$y(x^2+1) = \int 4x^2 dx = \frac{4}{3}x^3 + C$$

$$y = \frac{4}{3}\frac{x^3}{x^2+1} + \frac{C}{x^2+1}$$

Option (D) is not correct.