Q1. The shortest distance between the lines
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$ is

(a) $\frac{\sqrt{293}}{7}$
(b) 0
(c) $\frac{7}{\sqrt{293}}$
(d) $\frac{7}{\sqrt{293}}$

Q2. The value of λ, so that the vector $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ are perpendicular to each other, is:

(a) $\frac{5}{2}$

(b) $\frac{5}{4}$

(c) 5

(d) $\frac{7}{2}$

Q3. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (x^5 + x^3 + x + 2)\, dx$ is:
(a) 0
(b) 2
(c) 2π
(d) π

Q4. The minimum value of $\left(x^2 + \frac{250}{x}\right)$ is:
(a) 25
(b) 50
(c) 75
(d) 85

Q5. The derivative of $\sin(tan^{-1} e^{2x})$ with respect of x is:

(a) $\frac{2e^{2x} \sin(tan^{-1} e^{2x})}{1+e^{4x}}$
(b) $\frac{2e^{2x} \cos(tan^{-1} e^{2x})}{1+e^{4x}}$
(c) $\frac{2e^{2x} \sin(tan^{-1} e^{2x})}{1+e^{x^2}}$
(d) $\frac{2e^{2x} \cos(tan^{-1} e^{2x})}{1+e^{2x}}$

Q6. If B is a non-singular 4 × 4 matrix and A is its adjoint such that $|A| = 125$, then $|B|$ is
(a) 5
(b) 25
(c) 125

(d) 625

Q7. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$ then |3AB| is:
(a) – 9
(b) – 81
(c) – 27
(d) 81

Q8. Corner points of a feasible bounded region are (0, 10), (4, 2), (3, 7) and (10, 6). Maximum value 50 of objective function $z = ax +$ by occurs at two points (0, 10) and (10, 6). The value of a and b are:

(a) a = 5, b = 2
(b) a = 4, b = 5
(c) a = 2, b = 5
(d) a = 5, b = 4

Q9. The vertices of a closed convex polygon representing the feasible region of the LPP with objective function $z = 5x + 3y$ are (0, 0,) (3, 1), (1, 3) and (0, 2). The maximum value of z is

(a) 6
(b) 18
(c) 14
(d) 15

Q10. The general solution of the differential equation xdy + $(y - e^x)dx = 0$ is:
(a) $e^{xy} + e^x = C$, Where C is constant of integration

(b) $\frac{x^2}{2} + xy - e^x = C$, Where C is constant of integration

(c) $\frac{x^2}{2} + \frac{y^2}{2} - e^x = C$, Where C is constant of integration

(d) $xy - e^x = C$, Where C is constant of integration

Solutions:

S1. Ans. (a)

Sol. Given equations of lines are

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

Here

$\frac{2}{4} = \frac{3}{6} = \frac{6}{12} = \frac{1}{2}$. So, given lines are parallel.

Shortest distance between lines $= \left|\frac{\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\vec{b}|}\right|$

Here

$\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}, \overrightarrow{a_2} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$

$(\overrightarrow{a_2}-\overrightarrow{a_1}) = (3\hat{\imath}+3\hat{\jmath}-5\hat{k})-(\hat{\imath}+2\hat{\jmath}-4\hat{k}) = 2\hat{\imath}+\hat{\jmath}-\hat{k}$

$\vec{b} = 2\hat{\imath}+3\hat{\jmath}+6\hat{k}$

$|\vec{b}| = \sqrt{4+9+36} = 7$

Now, distance $= \left|\frac{(2\hat{\imath}+3\hat{\jmath}+6\hat{k})\times(2\hat{\imath}+\hat{\jmath}-\hat{k})}{7}\right|$

$= \left|\frac{-9\hat{\imath}+14\hat{\jmath}-4\hat{k}}{7}\right| = \frac{\sqrt{293}}{7}$

S2. Ans. (a)

Sol. Given vectors are

$\vec{a} = 2\hat{\imath}+\lambda\hat{\jmath}+\hat{k}$ and $\vec{b} = \hat{\imath}-2\hat{\jmath}+3\hat{k}$

The vectors are perpendicular, then

$\vec{a}.\vec{b} = (2\hat{\imath}+\lambda\hat{\jmath}+\hat{k}).(\hat{\imath}-2\hat{\jmath}+3\hat{k}) = 0$

$2-2\lambda+3 = 0$

$\lambda = \frac{5}{2}$

S3. Ans. (c)

Sol. Given

$\int_{\frac{-\pi}{2}}^{\pi/2} (x^5+x^3+x+2)\,dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^5+x^3+x)\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2dx$

$= 0 + \{2x\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ {Since $f(x) = x^5+x^3+x$ is an odd function}

$= 2\left\{\frac{\pi}{2}+\frac{\pi}{2}\right\} = 2\pi$

S4. Ans. (c)

Sol. Given function is

$f(x) = \left(x^2+\frac{250}{x}\right)$

$f'(x) = 2x-\frac{250}{x^2} = \frac{2x^3-250}{x^2}$

$f'(x) = 0$

$\frac{2x^3-250}{x^2} = 0$

$2x^3-250 = 0 \Rightarrow x^3 = 125 \Rightarrow x = 5$

Now minimum value $= (5)^2+\frac{250}{5} = 25+50 = 75$

S5. (b)

Sol. Given

$\sin(\tan^{-1} e^{2x})$

d. w. r. to x,

$\frac{d}{dx}[\sin(\tan^{-1} e^{2x})] = \cos(\tan^{-1} e^{2x}) \times \frac{d}{dx}(\tan^{-1} e^{2x})$

$= \cos(\tan^{-1} e^{2x}) \times \frac{1}{1+(e^{2x})^2} \times 2e^{2x}$

$= \frac{2e^{2x}\cos(\tan^{-1} e^{2x})}{1+e^{4x}}$

S6. Ans. (a)

Sol. Given

$A = adj(B)$ & $|A| = 125$

$|A| = |adj(B)|$

We have

$|adj(B)| = |B|^{4-1} = |B|^3$

$|A| = |B|^3$

$|B| = |A|^{\frac{1}{3}} = (125)^{\frac{1}{3}} = 5$

S7. Ans. (b)

Sol. Let A and B be square matrices of order 3 such that $|A| = -1, |B| = 3$

Now

$|3AB| = 3^3|A||B| = 27 \times (-1) \times 3 = -81$

S8. Ans. (c)

Sol. Given points are (0, 10), (4, 2), (3, 7) and (10, 6).

Also given maximum value of z is 50 which occurs at (0, 10) & (10, 6), then we have

$a(0) + b(10) = a(10) + b(6) = 50$

$10b = 10a + 6b = 50$

If $10b = 50$

$\Rightarrow b = 5$

$10a + 6b = 50 \Rightarrow 10a + 6 \times 5 = 50$

$10a = 20$

$a = 2$

S9. And. (b)

Sol. $z(0, 0) = 0$

$z(3, 1) = 5.3 + 3.1 = 18$

$z(1, 3) = 5.1 + 3.3 = 14$

$z(0, 2) = 5.0 + 3.2 = 6$

So z_{max} is 18 at (3, 1)

S10. Ans. (d)

Sol. Given

$xdy + (y - e^x)dx = 0$

$x\frac{dy}{dx} + y - e^x = 0$

$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

Integrating factor $= e^{\int P dx} = e^{\int \frac{dx}{x}} = e^{\log x} = x$

Solution is

$y \times x = \int \frac{e^x}{x} \times x \, dx = e^x + C$

$xy = e^x + C \Rightarrow xy - e^x = C$